NAG Toolbox for MATLAB

f08af

1 Purpose

f08af generates all or part of the real orthogonal matrix Q from a QR factorization computed by f08ae or f08be.

2 Syntax

```
[a, info] = f08af(a, tau, 'm', m, 'n', n, 'k', k)
```

3 Description

f08af is intended to be used after a call to f08ae or f08be, which perform a QR factorization of a real matrix A. The orthogonal matrix Q is represented as a product of elementary reflectors.

This function may be used to generate Q explicitly as a square matrix, or to form only its leading columns.

Usually Q is determined from the QR factorization of an m by p matrix A with $m \geq p$. The whole of Q may be computed by:

```
[a, info] = f08af(a, tau, 'k', p);
```

(note that the array **a** must have at least m columns) or its leading p columns by:

```
[a, info] = f08af(a(:,1:p), tau, 'k', p);
```

The columns of Q returned by the last call form an orthonormal basis for the space spanned by the columns of A; thus f08ae followed by f08af can be used to orthogonalise the columns of A.

The information returned by the QR factorization functions also yields the QR factorization of the leading k columns of A, where $k < p$. The orthogonal matrix arising from this factorization can be computed by:

```
[a, info] = f08af(a, tau, 'k', k);
```

or its leading k columns by:

```
[a, info] = f08af(a(:,1:p), tau, 'k', k);
```

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**lda**,$*$) – **double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

Details of the vectors which define the elementary reflectors, as returned by f08ae or f08be.

2: **tau**($*$) – **double array**

Note: the dimension of the array **tau** must be at least $\max(1, \mathbf{k})$.

Further details of the elementary reflectors as returned by f08ae or f08be.

5.2 Optional Input Parameters

1: **m – int32 scalar**

Default: The first dimension of the array **a**.

m, the order of the orthogonal matrix Q.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix Q.

Constraint: $\mathbf{m} \geq \mathbf{n} \geq 0$.

3: **k – int32 scalar**

Default: The dimension of the array **tau**.

k, the number of elementary reflectors whose product defines the matrix Q.

Constraint: $\mathbf{n} \geq \mathbf{k} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

5.4 Output Parameters

1: **a(lda,$*$) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix Q.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows: 1: **m**, 2: **n**, 3: **k**, 4: **a**, 5: **lda**, 6: **tau**, 7: **work**, 8: **lwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that

$$\|E\|_2 = O(\epsilon),$$

where ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $4mnk - 2(m+n)k^2 + \frac{4}{3}k^3$; when $n = k$, the number is approximately $\frac{2}{3}n^2(3m-n)$.

The complex analogue of this function is f08at.

9 Example

```
a    =    [3.61767881382524,    -0.5565999923223895,    0.847366545721238,
0.7460032078266114;
           0.4608758421558694,  -2.028077032202356,  0.5513872350020937,
1.16996276895585;
        -0.5492302782168392, -0.04571098289280237, 1.374460641222295, -
1.410473781059997;
         0.4608758421558694, 0.2828431690617352, 0.004430814804361739, -
2.375527319588618;
       -0.03581936597066342, 0.0796426824688576, -0.07728561757441148, -
0.5213744847432364;
           0.004775915462755124,  0.3002942085609617,  0.801665355572228,
0.2558113872182322];
tau = [1.157559592582321;
     1.696915139470381;
     1.213106371299621;
     1.495583371241627];
[aOut, info] = f08af(a, tau)
```

```
aOut =
   -0.1576    0.6744   -0.4571    0.4489
   -0.5335   -0.3861    0.2583    0.3898
    0.6358   -0.2928    0.0165    0.1930
   -0.5335   -0.1692   -0.0834   -0.2350
    0.0415   -0.1593    0.1475    0.7436
   -0.0055   -0.5064   -0.8339    0.0335
info =
           0
```